PUMPS AND FANS–BASIC LAWS

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The basic pump and fan laws are derived using the principles of dynamic similarity and dimensional analysis. For our purposes here the results of this analysis may be presented as two dimensionless parameters from which the basic pump and fan laws may be derived. These are:

$$Q/ND^3 = \text{constant} = c_1 \quad H/N^2D^2 = \text{constant } c_2 \tag{10-1}$$

where Q = volumetric flow rate, gal/min
N = rotative speed, rpm
D = impeller diameter, ft
$H = \Delta p/\rho$ = head developed, ft
$\Delta\rho$ = pressure rise, lb/ft^2
ρ = density, lb/ft^3

The horsepower required is

$$\text{hp} = \frac{(\text{flow rate})\,\text{lb/min} \times (\text{head developed})\,\text{ft}}{(\text{efficiency})\,(33{,}000\text{ ft-lb/min-hp})}$$

$$= \frac{\rho QH}{33{,}000(\eta)} \tag{10-2}$$

where η = efficiency.

Using the dimensionless parameters of Eq. (10-1) in the horsepower expression

$$\text{hp} = \frac{\rho(c_1ND^3)c_2N^2D^2}{\eta(33{,}000)} \quad \text{or} \quad \frac{\text{hp}}{\rho N^3D^5} = c_3 \tag{10-3}$$

as long as the efficiency remains constant.

From Eqs. (10-1) and (10-3) we can deduce the standard pump and fan laws. These are presented below for convenience. It is essential to realize that these laws only hold true at differing operating conditions as long as the pump or fan *efficiency* is constant.

A. *Variation in Impeller Speed*
Constant impeller size, density, system
1. Q varies as fan speed
2. H varies as square of fan speed
3. Power varies as cube of fan speed

B. *Variation in Impeller Size*
Constant tip speed (ND), density, same proportions
1. Q varies as D^2
2. H remains constant
3. N varies as 1/D
4. Power varies as D^2

C. *Variation in Impeller Size*
$N = c$, $\rho = c$, same proportions
1. Q varies as D^3
2. H varies as D^2
3. Tip speed varies as D
4. Power varies as D^5

D. *Variation in Density*
$Q = c$, system $= c$, $N = c$, $D = c$
1. H varies as ρ
2. Power varies as ρ

E. *Variation in Density*
$\Delta p = c$, system $= c$, $D = c$
1. Q varies as $\sqrt{1/\rho}$
2. N varies as $\sqrt{1\rho}$
3. Power varies as $\sqrt{1/\rho}$

F. *Variation in Density*
Weight flow $= c$, system $= c$, $D = c$
1. Q varies as $1/\rho$
2. Head varies as $1/\rho$
3. N varies as $1/\rho$
4. Power varies as $1/\rho^2$

In each of the above groups where system was specified as constant, this means that the piping system or duct system remains unchanged.

Problem 10-1 Capacity, Static Pressure, and Horsepower of Fan

A fan of 36-in. wheel diameter is to handle 24.000 cu ft/min of standard air at 810 rpm with a static pressure of 3.25 in. H_2O and requires 14.8 bhp. If a 48-in. fan of the same homologous series, operating at the same speed, handles air at 0.060 lb/cu ft density, determine: (a) capacity, (b) static pressure, and (c) horsepower.

Solution:

The basic fan parameters Q/ND^3 and H/N^2D^2 remain constant.

(a) The change in diameter results in a new value of Q:

$$\left(\frac{Q}{ND^3}\right)_{\text{new}} = \left(\frac{Q}{ND^3}\right)_{\text{exist}}$$

$$Q_{\text{new}} = 24{,}000\left(\frac{48}{36}\right)^3 = 56{,}900 \text{ cu ft/min} \quad \textit{Answer}$$

(b) Since the head $H = \dfrac{\Delta p\,(\text{lb/ft}^2)}{\rho\,(\text{lb/ft}^3)}$

$$\left(\frac{\Delta p}{\rho N^2D^2}\right)_{\text{old}} = \left(\frac{\Delta p}{\rho N^2D^2}\right)_{\text{new}}$$

$$\Delta p_{\text{new}} = 3.25\left(\frac{0.06}{0.075}\right)\left(\frac{48}{36}\right)^2 = 4.63 \text{ in. } H_2O$$

Answer

where 0.075 is the air density at standard conditions and the conversion factors from inches water to pounds per square foot would be the same on both sides of the equation and cancel out.

(c) $\text{hp} = \rho QH = Q\Delta p$

$$\frac{\text{hp}_{\text{new}}}{\text{hp}_{\text{old}}} = \frac{(Q\Delta p)_{\text{new}}}{(Q\Delta p)_{\text{old}}} = \frac{56{,}900(4.63)}{24{,}000(3.25)} = 3.37$$

$$\text{hp}_{\text{new}} = 3.37(14.8) = 49.7 \text{ hp} \quad \textit{Answer}$$

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Problem 10-2 Induced Draft Fan Requirements for Boiler

A boiler to generate 450,000 lb/hr of steam has just been installed near Boston, Mass., barometer 30 in. Hg. The acceptance test shows that the induced draft fan handles 600,000 lb/hr of flue gas at 290F against a suction of 10 in. H_2O. This suction can be considered as that necessary to overcome friction within the boiler gas passages, the air heater, the dust catcher, and the duct work.

This boiler is to be duplicated at a location near Denver, Colo., barometer 24.0 in. Hg. No changes will be made in any equipment in the induced draft circuit. However, the induced draft wheel will be adjusted to suit the elevation of the new location.

Assume the gas weight and temperature at Denver and Boston to be identical. What are the new volume and suction for which the fan must be designed and why?

Solution:

The weight flow = 600,000 lb/hr.

$$W = pV/RT$$

$$p = 30 \text{ in. Hg} \times 0.491 \times 144 - \frac{10 \text{ in. } H_2O}{12} \times 62.4$$

$$= 2068 \text{ lb/ft}^2$$

The volumetric flow in Boston is, assuming R for flue gases as 55,

$$V_B = \frac{55(460 + 290)(600{,}000)}{2068(60)} = 200{,}000 \text{ cu ft/min}$$

Since the weight flow and temperature in Denver are to be the same as in Boston, $p_B V_B = p_D V_D$. The head loss (in. H_2O) due to friction will be proportional to the velocity squared. The fan pressure will not be much different from barometric pressure. For example, in Boston, barometric pressure is 2120 lb/ft² as against 2068 lb/ft² for fan suction (14.73 psi against 14.3 psi).

As a good approximation to the answer we can assume the fan suction pressure in Denver = barometric pressure = 24 in. Hg.

Then the volumetric flow in Denver is

$$\frac{V_D}{200{,}000} = \frac{2068}{24(0.491)(144)}$$

$$V_D = 244{,}000 \text{ cu ft/min}$$

The head loss can then be computed:

$$\frac{(\text{Head loss})_D}{(\text{Head loss})_B} = \frac{V_D^2}{V_B^2}$$

For same weight flow,

$$\rho_B A_B V_B = \rho_D A_D V_D$$

$$\frac{p_B V_B}{RT} = \frac{p_D V_D}{RT}$$

$$\frac{(\text{Head loss})_D}{(\text{Head loss})_B} = \left(\frac{p_B}{p_D}\right)^2 = \frac{(2068)^2}{[(24)(0.491)(144)]^2}$$

$$= 1.48$$

$$(\text{Head loss})_D = 1.48(10) = 14.8 \text{ in. } H_2O$$

To check on the effect of the assumption that fan suction pressure is approximately barometric, we can use this answer to recompute our result.

Fan suction pressure in Denver = 24(0.491)(144) − (14.8/12)(62.4) = 1623 lb/ft²

This would change the volumetric flow to

$$V_D = \frac{2068(200{,}000)}{1623} = 255{,}000 \text{ cu ft/min}$$

and

$$(\text{Head loss})_D = \left(\frac{2068}{1623}\right)^2 (10) = 16.2 \text{ in. } H_2O$$

Recomputing the volumetric flow now would not change the numerical result appreciably.

Problem 10-3 Head and Flow Rate of Centrifugal Water Pump

A model centrifugal pump with a 3-in. diameter impeller delivers 600 gal/min of 60°F water at a total head of 350 ft when operating at 1750 rpm. Find the diameter of a geometrically similar pump that will deliver 1000 gpm when operating at 3500 rpm. What will be the total head of the 3500-rpm pump when it is delivering 1000 gal/min?

Solution:

$$(Q/ND^3)_1 = (Q/ND^3)_2$$

$$D_2^3 = (3)^3(1000/600)(1750/3500)$$

$$D_2 = 2.82 \text{ in.} \quad \textit{Answer}$$

$$\left(\frac{H}{N^2D^2}\right)_2 = \left(\frac{H}{N^2D^2}\right)_1$$

$$H_2 = 350\left(\frac{3500}{1750}\right)^2\left(\frac{2.82}{3}\right)^2 = 1242 \text{ ft} \quad \textit{Answer}$$

Problem 10-4 Horsepower, Speed, and Capacity of Electrically Driven Water Pump

A dc motor-driven pump running at 100 rpm delivers 500 gal/min of water against a total pumping head of 90 ft with a pump efficiency of 60%. (a) What motor horsepower is required? (b) What speed and capacity would result if the pump rpm were increased to produce a pumping head of 120 ft assuming no change in efficiency? (c) Can a 25-hp motor be used under conditions indicated by (b)?

Solution:

(a) $$\text{Pump hp} = \frac{500 \text{ gal/min} \times 8.33 \text{ lb/gal} \times 90 \text{ ft}}{0.6(33{,}000)}$$

$$= 19 \text{ hp} \quad \textit{Answer}$$

(b) $H/N^2D^2 = \text{const.} \quad 120/N^2/D^2 = 90/100^2D^2$

$$N = 115 \text{ rpm} \quad \textit{Answer}$$

(c) $hp = \rho QH$

$$hp_{new} = hp_{old}\frac{(QH)_{new}}{(QH)_{old}}$$

$$\left(\frac{Q}{ND^3}\right)_{new} = \left(\frac{Q}{ND^3}\right)_{old}$$

$$Q_{new} = 500\left(\frac{115}{100}\right) = 575 \text{ gal/min}$$

$$hp_{new} = 19\left(\frac{575}{500}\right)\left(\frac{120}{90}\right) = 29.1 \quad \textit{Answer}$$

ΔΔΔ

NOTES ON AXIAL FANS

1. INLET CONDITIONS CAN BE CRITICAL. WATCH THEM.

2. NOISE NEED NOT BE THE PROBLEM NOW THAT IT WAS PREVIOUSLY. DURING THE PAST FIVE YEARS OUR INDUSTRY HAS HAD A MAJOR BREAK THROUGH IN THE MEASUREMENT AND PREDICTION OF NOISE FROM AIR MOVING EQUIPMENT. WITH THIS NEW POWER LEVEL DATA WE CAN NOW PREDICT FAIRLY ACCURATELY RESULTANT NOISE LEVELS FROM AIR HANDLING SYSTEMS. A WELL DESIGNED AXIAL IS INHERENTLY NOISIER THAN A WELL DESIGNED CENTRIFUGAL - APPROXIMATELY 6-8 DECIBELS. HOWEVER, WITH THIS NEW ENGINEERING DATA AND SILENCERS CAN BE ADDED TO REDUCE NOISE LEVELS FROM AXIAL SYSTEMS WITHIN DESIGNED NOISE CRITERIA.

3. BASIC SELECTION. USE DESIGN CAPACITIES (CFM) AND APPROXIMATE CALCULATED PRESSURES AND SELECT FAN SIZE IN OPTIMUM RANGE, ADD SAFETY FACTORS TO THE MOTOR SIZE. BE CONSCIOUS OF WHERE THE PRESSURE BREAK RANGE IS ON THE PARTICULAR AXIAL FAN DESIGN THAT YOU ARE USING. FOR BELT DRIVEN FANS THERE IS ONE OPTIMUM SIZE FAN OF A PARTICULAR DESIGN FOR EACH AIR SYSTEM - NO MATTER HOW MUCH AIR THIS SYSTEM IS TO HANDLE.

   IT IS MORE DIFFICULT TO SELECT A DIRECT DRIVE FAN IN THE OPTIMUM RANGE BUT IT CAN BE DONE.

4. THERE IS AN AXIAL FORCE IN THE OPPOSITE DIRECTION TO AIR FLOW (5.19 x TOTAL PRESSURE x FAN INLET AREA IN LBS.). FANS WITH INLET AND OUTLET FLEXIBLE CONNECTION REQUIRE STABLIZERS AT HIGHER PRESSURES TO PREVENT FANS FROM MOVING AXIALLY.

5. MOTOR STARTING TORQUE IS NOT A PROBLEM LIKE IT IS ON LARGE LOW SPEED CENTRIFUGALS.

6. FOR AXIALS USE TOTAL PRESSURE, NOT STATIC PRESSURE IN SELECTION.

7. INLET CONDITIONS ARE CRITICAL FOR AXIALS. IF YOU MUST USE AN ELBOW PUT IN ON THE FAN OUTLET. IF IT MUST BE PUT ON FAN INLET USE SPECIAL ELBOWS DESIGNED BY THE FAN MANUFACTURER, OR LONG RADIUS ELBOWS WITH A FAN WHEEL DIAMETER OF STRAIGHT DUCT ON THE INLET; OR A 90° ELBOW WITH TURNING VANES AND A STRAIGHT SECTION TWICE THE WHEEL DIAMETER ON THE INLET SIDE.

8. USE ONLY STRONG HEAVY DUTY DAMPERS (MOTORIZED OR BACK-DRAFT) ON AXIAL FAN DISCHARGES. MOUNT DAMPERS AS FAR FROM FAN DISCHARGE AS POSSIBLE.

9. AN AIR SWIRL AT FAN INLET DRASTICALLY CHANGES THE FAN PERFORMANCE CURVE. ASSUMING A PRESSURE LOSS IS NO SOLUTION. USE INLET VORTEX BREAKERS.

10. ON OPEN INLET AXIALS USE INLET BELLS.

11. AXIAL FANS TAKE EXCESSIVE AMOUNT OF HORSEPOWER NEAR AND AT SHUT-OFF CONDITIONS. DAMPER ONLY IN SELECTION RANGE.

12. LIMIT LOAD BRAKE HORSEPOWER USUALLY REFERS TO MAXIMUM BRAKE HORSEPOWER IN SELECTION RANGE. BHP NEAR SHUT-OFF IS USUALLY MUCH HIGHER THAN THE LIMIT LOAD BHP.

13. MOST AXIAL FANS HAVE RELATIVELY STEEP PRESSURE CURVES - FOR MOST SYSTEMS THIS IS GOOD.